

EXO 1

$$(a) \begin{cases} 2x - 3y + z = -2 \\ x + 2y - z = 3 \end{cases} \Leftrightarrow \begin{cases} x + 2y - z = 3 & (L2) \\ -7y + 3z = -8 & (L1) - 2(L2) \end{cases} : \text{COMPATIBLE}$$

$$(b) \begin{cases} 2x - 3y + z + t = 0 \\ x + 2y - z - 3t = 0 \\ -x - y + 2t = 0 \end{cases} \text{ est homogène donc COMPATIBLE}$$

$$(c) \begin{cases} x + 2y = 3 \\ 0 = 1 \end{cases} \text{ est INCOMPATIBLE car } 0 \neq 1 \text{ (di, je vous jure!)}$$

EXO 2

$$(a) \begin{cases} x - y = -1 \\ x + y = 1 \\ y = 2 \end{cases} \Leftrightarrow \begin{cases} x = y - 1 = 1 \\ x = 1 - y = -1 \\ y = 2 \end{cases} \Leftrightarrow \begin{cases} x = 1 \\ y = 2 \\ -1 = 1 \end{cases} \text{ INCOMPATIBLE}$$

$$(b) \begin{cases} x - y = -1 \\ 2x - 4y = -2 \\ -x + y = 1 \end{cases} \Leftrightarrow \begin{cases} -x + y = 1 & (L3) \\ 0 = 0 & (L1) + (L3) \\ 0 = 0 & (L2) + 2(L3) \end{cases} \text{ COMPATIBLE}$$

$$(c) \begin{cases} x + y + z = 1 \\ x - y - z = -1 \end{cases} \Leftrightarrow \begin{cases} x + y + z = 1 & (L1) \\ -2y - 2z = -2 & (L2) - (L1) \end{cases} \text{ COMPATIBLE}$$

EXO 3

$$(a) \begin{cases} x + 2y = 3 \\ x + 3y = 5 \end{cases} \Leftrightarrow \begin{cases} x + 2y = 3 & (L1) \\ y = 2 & (L2) - (L1) \end{cases} \Leftrightarrow \begin{cases} x = 3 - 4 = -1 \\ y = 2 \end{cases}$$

$$S = \{(-1; 2)\}$$

$$(b) \begin{cases} x+y+z=3 \\ -x+2y-z=-9 \\ 2x-y+3z=16 \end{cases} \Leftrightarrow \begin{cases} x+y+z=3 & (L1) \\ 3y=-6 & (L2)+(L1) \\ -3y+z=10 & (L3)-2(L1) \end{cases} \Leftrightarrow \begin{cases} x+y+z=3 \\ y=-2 \\ z=10-6=4 \end{cases}$$

$$\Leftrightarrow \begin{cases} x=3+2-4=1 \\ y=-2 \\ z=4 \end{cases}$$

$$\mathcal{S} = \{(1; -2; 4)\}$$

$$(c) \begin{cases} x+2y-z+t=3 \\ 2x+4y+z-2t=-1 \end{cases} \Leftrightarrow \begin{cases} x+2y-z+t=3 & (L1) \\ 3z-4t=-7 & (L2)-2(L1) \end{cases}$$

4 inconnues
Rang = 2
↓
2 paramètres...
on prend y et t

$$\Leftrightarrow \begin{cases} x-z=3-2y-t \\ 3z=-7+4t \end{cases}$$

$$\Leftrightarrow \begin{cases} x-z=3-2y-t \\ z=-\frac{7}{3}+\frac{4}{3}t & (L2)/3 \end{cases}$$

$$\Leftrightarrow \begin{cases} x=\frac{2}{3}-2y+\frac{1}{3}t & (L1)+(L2) \\ z=-\frac{7}{3}+\frac{4}{3}t & (L2) \end{cases}$$

d'où

$$\mathcal{S} = \left\{ \left(\frac{2}{3}-2y+\frac{1}{3}t, y, -\frac{7}{3}+\frac{4}{3}t, t \right) / y \in \mathbb{R} \text{ et } t \in \mathbb{R} \right\}$$

Exo 4

$$(a) \begin{cases} x-3y+2z=2 \\ 2y-z=-1 \end{cases} \Leftrightarrow \begin{cases} x+2z-3y=2 \\ 3-2y=1 \end{cases}$$

3 inconnues
rang = 2 $\Rightarrow$ 1 paramètre
... y ici

$$\Leftrightarrow \begin{cases} x+2z=2+3y \\ z=1+2y \end{cases} \Leftrightarrow \begin{cases} x=-y & (L1)-2(L2) \\ z=1+2y & (L2) \end{cases}$$

d'où

$$\mathcal{S} = \{ (-y, y, 1+2y) / y \in \mathbb{R} \}$$

(droite de $\mathbb{R}^3$ cf chapitre XIV)

$$(b) \begin{cases} 3x+2y-z=-1 \\ 2x-y=-4 \\ x+y+z=3 \end{cases} \Leftrightarrow \begin{cases} x+y+z=3 & (L3) \\ -y-4z=-10 & (L1)-3(L3) \\ -3y-2z=-10 & (L2)-2(L3) \end{cases}$$

$$\Leftrightarrow \begin{cases} x+y+z=3 \\ y+4z=10 & -(L2) \\ 10z=20 & (L3)-3(L2) \end{cases} \Leftrightarrow \begin{cases} x=3-4=-1 \\ y=10-8=2 \\ z=2 \end{cases}$$

$$\mathcal{S} = \{(-1; 2; 2)\}$$

$$(c) \begin{cases} 2y-z=2 \\ 2x-3y+z=-5 \\ x+y+z=0 \\ x+2y-z=1 \end{cases} \Leftrightarrow \begin{cases} x+y+z=0 & (L3) \\ 2y-z=2 & (L1) \\ -5y-z=-5 & (L2)-2(L3) \\ y-2z=1 & (L4)-(L3) \end{cases}$$

$$\Leftrightarrow \begin{cases} x+y+z=0 \\ y-2z=1 & (L4) \\ 3z=0 & (L2)-2(L4) \\ -11z=0 & (L3)+5(L4) \end{cases} \Leftrightarrow \begin{cases} x=-1 \\ y=1 \\ z=0 \end{cases}$$

$$\mathcal{S} = \{(-1; 1; 0)\}$$

Exo 5

$$(a) \begin{cases} x+2y-z+k=1 \\ x+3y+z-k=2 \\ -x+y+7z+2k=3 \\ 2x+y-8z+k=4 \end{cases} \Rightarrow \begin{cases} x+2y-z+k=1 & (L1) \\ y+2z-2k=1 & (L2)-(L1) \\ 3y+6z+3k=4 & (L3)+(L1) \\ -3y-6z-k=2 & (L4)-2(L1) \end{cases} \Rightarrow \begin{cases} x+2y-z+k=1 \\ y+2z-2k=1 \\ 9k=1 & (L3)-3(L2) \\ 2k=6 & (L4)+(L3) \end{cases}$$

SYSTÈME INCOMPATIBLE q. on obtient $k = \frac{1}{9} \neq 3$

$$(b) \begin{cases} x+y-3z-4t = -1 \\ 2x+2y+2z-3t = 2 \\ 3x+6y-2z+t = 8 \\ 2x+y+5z+t = 5 \end{cases} \Leftrightarrow \begin{cases} x+y-3z-4t = -1 \\ 8z+5t = 4 \quad (L2)-2(L1) \\ 3y+7z+13t = 11 \quad (L3)-3(L1) \\ -y+11z+9t = 7 \quad (L4)-2(L1) \end{cases} \Leftrightarrow \begin{cases} x+y-3z-4t = -1 \\ -y+11z+9t = 7 \quad (L4) \\ 8z+5t = 4 \quad (L2) \\ 40z+40t = 32 \quad (L3)+3(L4) \end{cases}$$

$$\Leftrightarrow \begin{cases} x+y-3z-4t = -1 \\ -y+11z+9t = 7 \\ 8z+5t = 4 \\ 5z+5t = 4 \quad (L4)/8 \end{cases} \Leftrightarrow \begin{cases} x+y-3z-4t = -1 \\ -y+11z+9t = 7 \\ 5z+5t = 4 \quad (L4) \\ 3z = 0 \quad (L3)-(L4) \end{cases} \Leftrightarrow \begin{cases} z = 0 \\ t = 4/5 \\ y = 36/5 - 7 = 1/5 \\ x = -1 - 1/5 + 16/5 = 10/5 = 2 \end{cases}$$

$$\mathcal{S} = \{(2; 1/5; 0; 4/5)\}$$

$$(c) \begin{cases} x-y+z+t = -1 \\ 5x+2y-z+3t = 5 \\ -3x-4y+3z+2t = 1 \\ 6x+y-2t = 8 \end{cases} \Leftrightarrow \begin{cases} x-y+z+t = -1 \\ 7y-6z-2t = 10 \quad (L2)-5(L1) \\ -7y+6z+5t = -2 \quad (L3)+3(L1) \\ -7y+6z+2t = 10 \quad (L4)+2(L3) \end{cases}$$

donc
 $0 = 20 \quad (L2)+(L4)$
SYSTEME INCOMPATIBLE!

$$\mathcal{S} = \emptyset$$

EXO 6

$$(a) \begin{cases} x-2y+3z = 5 \\ 2x-4y+z = 5 \\ 3x-5y+2z = 8 \end{cases} \Leftrightarrow \begin{cases} x-2y+3z = 5 \\ -5z = -5 \quad (L2)-2(L1) \\ y-7z = -7 \quad (L3)-3(L1) \end{cases} \Leftrightarrow \begin{cases} x-2y+3z = 5 \\ y-7z = -7 \\ z = 1 \quad (L2)/(-5) \end{cases}$$

$$\Leftrightarrow \begin{cases} x = 2 \\ y = 0 \\ z = 1 \end{cases} \quad \mathcal{S} = \{(2; 0; 1)\}$$

$$(b) \begin{cases} x+2y-z = 5 \\ 2x+y+z = 10 \\ x+2z = 0 \end{cases} \Leftrightarrow \begin{cases} x+2z = 0 \\ 2y-3z = 5 \quad (L1)-(L3) \\ y-3z = 10 \quad (L2)-2(L3) \end{cases} \Leftrightarrow \begin{cases} x+2z = 0 \\ -3z+4y = 5 \\ y = -5 \quad (L2)-(L3) \end{cases}$$

$$\Leftrightarrow \begin{cases} x = 10 \\ z = (2y-5)/3 = -5 \\ y = -5 \end{cases} \quad \mathcal{S} = \{(10; -5; -5)\}$$

$$(c) \begin{cases} x - y + 3z = 2 \\ -x + 4y + z = -1 \\ 2x - 2y - 3z = 4 \end{cases} \Leftrightarrow \begin{cases} x - y + 3z = 2 \\ 3y + 4z = 1 & (L2) + (L1) \\ -9z = 0 & (L3) - 2(L1) \end{cases} \Leftrightarrow \begin{cases} x = 2 + 1/3 = 7/3 \\ y = 1/3 \\ z = 0 \end{cases}$$

$$\mathcal{S} = \{(7/3; 1/3; 0)\}$$

$$(d) \begin{cases} 2x + y - z = 3 \\ x - y + z = 2 \\ x + y + 2z = 0 \end{cases} \Leftrightarrow \begin{cases} x - y + z = 2 \\ 3y - 3z = -1 & (L1) - 2(L2) \\ 2y + z = -2 & (L3) - (L2) \end{cases} \Leftrightarrow \begin{cases} x + z - y = 2 \\ z + 2y = -2 \\ 9y = -7 & (L2) + 3(L3) \end{cases}$$

$$\Leftrightarrow \begin{cases} x = 2 + 4/9 - 7/9 = 15/9 \\ z = -2 + 14/9 = -4/9 \\ y = -7/9 \end{cases}$$

$$\mathcal{S} = \{(15/9; -7/9; -4/9)\}$$

EX07

$$(a) \begin{cases} -y + z = 1 \\ -5x + 2y - z = -1 \\ x - 2z = 4 \end{cases} \Leftrightarrow \begin{cases} x - 2z = 4 \\ -y + z = 1 \\ 2y - 11z = 19 & (L2) + 5(L3) \end{cases} \Leftrightarrow \begin{cases} x - 2z = 4 \\ -y + z = 1 \\ -9z = 21 & (L3) + 2(L2) \end{cases}$$

$$\Leftrightarrow \begin{cases} x = 2z + 4 = -2/3 \\ y = z - 1 = -10/3 \\ z = -7/3 \end{cases}$$

$$\mathcal{S} = \{(-2/3; -10/3; -7/3)\}$$

$$(b) \begin{cases} y - 2z = 3 \\ -2x - 3y + z = 2 \\ 3x + y - 2z = 0 \end{cases} \Leftrightarrow \begin{cases} 3x = -3 & (L3) - (L1) \\ y - 2z = 3 \\ -2x - 3y + z = 2 \end{cases} \Leftrightarrow \begin{cases} x = -1 \\ y - 2z = 3 \\ -3y + z = 0 & (L3) + 2(L1) \end{cases}$$

$$\Leftrightarrow \begin{cases} x = -1 \\ y - 2z = 3 \\ -5z = 9 & (L3) + 3(L2) \end{cases} \Leftrightarrow \begin{cases} x = -1 \\ y = 3 + 2z = -3/5 \\ z = -9/5 \end{cases}$$

$$\mathcal{S} = \{(-1; -3/5; -9/5)\}$$

$$(c) \begin{cases} 10x + 9y + z = -50 \\ 9x + 10y + 5z = 40 \\ x + 5y + 9z = 180 \end{cases} \Rightarrow \begin{cases} x + 5y + 9z = 180 & (L3) \\ -41y - 89z = -1850 & (L1) - 10(L3) \\ -35y - 76z = -1580 & (L2) - 9(L3) \end{cases}$$

$$\begin{cases} x + 5y + 9z = 180 \\ 35y + 76z = 1580 & -(L3) \\ z = 30 & 35(L2) - 41(L3) \end{cases} \Rightarrow \begin{cases} x = 180 + 100 - 270 = 10 \\ y = (1580 - 76 \times 30) \div 35 = -20 \\ z = 30 \end{cases}$$

$$\mathcal{Y} = \{(10; -20; 30)\}$$

$$(d) \begin{cases} x + 2y - 3z = 4 \\ x + 3y - z = 11 \\ 2x + 5y - 5z = 13 \\ x + 4y + z = 18 \end{cases} \Rightarrow \begin{cases} x + 2y - 3z = 4 \\ y + 2z = 7 & (L2) - (L1) \\ y + z = 5 & (L3) - 2(L1) \\ 2y + 4z = 14 & (L4) - 2(L1) \end{cases} \Rightarrow \begin{cases} x + 2y - 3z = 4 \\ y + z = 5 & (L3) \\ z = 2 & (L2) - (L3) \\ 0 = 0 & (L4) - 2(L2) \end{cases}$$

$$x = 4 - 6 + 6 = 4$$

$$\Rightarrow y = 5 - 2 = 3 \\ z = 2$$

$$\mathcal{Y} = \{(4; 3; 2)\}$$

$$(e) \begin{cases} y + z = 5 \\ x + z = 4 \\ x + y + 2z = 9 \\ -x + y = 1 \end{cases} \Rightarrow \begin{cases} x + z = 4 & (L2) \\ y + z = 5 & (L4) + (L2) \\ y + z = 5 & (L3) - (L2) \\ y + z = 5 & (L1) \end{cases} \Rightarrow \begin{cases} x = 4 - z \\ y = 5 - z \\ z \in \mathbb{R} \end{cases}$$

$$\mathcal{Y} = \{(4 - z; 5 - z; z) \text{ avec } z \in \mathbb{R}\}$$

$$\begin{aligned}
 (\neq) \quad \begin{cases} 4x + 8z = 1 \\ y - 6z = 2 \\ 2x - 4z = 3 \\ 5x - y + z = 2 \end{cases} &\Rightarrow \begin{cases} x - 2z = 3/2 & (L3)/2 \\ 16z = -5 & (L1) - 2(L3) \\ y - 6z = 2 \\ -y + 11z = -11/2 & (L4) - \frac{5}{2}(L3) \end{cases} \Rightarrow \begin{cases} x - 2z = 3/2 \\ y - 6z = 2 \\ 16z = -5 \\ 5z = -7/2 & (L4) + (3) \end{cases}
 \end{aligned}$$

$$\boxed{y = \emptyset}$$

$$z = -\frac{5}{16} = -\frac{7}{10} \Rightarrow \text{CONTRADICTION}$$

C'est donc un système incompatible

EX08

$\Rightarrow$ mêmes méthodes !
 J'en ai marre de ces #e#!? de systèmes

SUITE DANS LA SECONDE PARTIE